How Many Dimensions?

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It is currently the fashion, both in "popular" and academic articles on cosmology, to resort to more than the four dimensions of mere spacetime in order to formulate "theories of everything". According to a newsletter from one of America's most prestigious universities, "string theory requires that space-time contain up to seven extra dimensions if it is to include gravity. Extra dimensions could also help solve two cosmological mysteries: What were the initial conditions of the universe and what is the mysterious dark energy that is accelerating the expansion of the universe?". Artists depict in two dimensions "multiverses" of disjoint "universes". Those cosmologists habituated to the four dimensions of spacetime and the idea of its curvature as the coin of dynamics see such a generalization as a necessary extension of theory; anything less is simply inadequate for a truly comprehensive representation of the real world.

Perhaps. Whether or not this is so, it would seem to behoove the proponents of such a proposition to provide pertinent definitions by which skeptics and unbelievers could be persuaded. First and foremost would surely be a definition of the term "dimension". A mathematician can airily add new variables to his description of a dynamical system, but any such new variables would at some stage have to be identified with a physically measurable quantity. Would such a quantity automatically qualify as an additional dimension?

To narrow the inquiry, let us consider those quantities which we all agree are indeed "dimensions". Of the four dimensions of spacetime, consider the time dimension. Time is defined as that which is measured by a clock. Before dismissing this as a simplistic evasion, let us define a clock. A generalized clock may be defined as ***a monotonic single-valued parametrization of the sequence of events at a single observer***. This means that each event of a continuous sequence of events witnessed by a single observer can be assigned a real number such that later events have assigned to them larger numbers. The earlier-later relation of events is put into a one-to-one correspondence with the smaller-larger relation of some segment of the real numbers. Since the latter has no gaps and no discontinuities, neither does the former. A wall clock or a wrist watch has hands whose positions define an unbroken numerical sequence of times. This is true even if the watch or clock runs fast or slow. We disallow the watch to run backwards, however.

The critical feature of a clock is that it offers a continuous measure of a quantity for which there is a continuous ordering relation. The same may be said for each of the three spatial dimensions. An object can be at a greater or lesser distance, more or less to the right, higher or lower. Clearly, the continuum of locations should be possible to be parametrized with some segment of the real numbers, a segment of non-zero measure for each dimension. This precludes parametrizing with the integers or merely with the rational numbers, both of which, though infinite, are of measure zero.

Therefore, the first question we should ask of any new dimension is: What is its defining order relation? Let us start with a fifth dimension. If a fifth dimension can be defined by some ordering relation, we can then proceed to seek a sixth dimension, then a seventh, ..., and so on, to as many dimensions as can be identified by some ordering relation. If not, we are at an impasse. If there is no fifth dimension, there obviously can be no sixth, seventh, ..., twelfth.

So how do matters stand with dimensions higher than the fourth? A search for a fifth ordering relation has so far been a futile one. Not temperature, not entropy, not mass, not electric charge, not kinetic energy. All these can be evaluated using only the four dimensions of spacetime.

Does this mean that attempts to identify "dark matter" and "dark energy" are doomed? No. Dark matter, the presumed cause of the high velocity of stars on the outskirts of our own Milky Way Galaxy, can be accounted for as the mass-energy of a large assemblage of masses, each drawn from infinity by the gravitational attraction of the rest. Dark energy is the kinetic energy associated with this invisible dark matter (see "Cosmology in Flat Space-Time").

This leads us to the matter of "multiverses" — disjoint universes unknown to each other because they are incapable of intercommunication. Their number is said to be infinite and from this fact is drawn the conclusion that there must be at least one other universe identical to this one. Such a conclusion is not warranted, partly because discrete universes are countable ("denumerable") and the measure of them is therefore zero. The dimensions of such universes must be defined by ordering relations distinct from those of our spacetime. What are these disjoint ordering relations? So far, they have defied identification. The article in the aforementioned newsletter confesses that "scientists do not have direct evidence of extra dimensions at this time." Until such time as extra dimensions have been identified, we may dismiss both (a) dimensions higher than the fourth and (b) universes other than this one.

Note further that a dimension does not have a magnitude. It is sometimes averred that higher dimensions are "small", covertly coiled where they are easily overlooked. Are either distance or time "large or "small"?

October 1, 2008