The Mystery of Xi Persei

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One of the first triumphs of general relativity was the correct prediction of the advance of the perihelion of Mercury, inexplicable by Newtonian theory of gravitation. The observed amount was 574 seconds of arc per century, of which all but 43 seconds could be accounted for. Einstein's new general theory was the only theory to account for the full amount. The same equations which gave Mercury's advance of perihelion correctly were later applied to the motion of binary star systems, but with mixed results; in some cases the predicted advance was too great, in others too small. One of these aberrant systems is the star ξ (xi) Persei. What is the explanation?

To answer this question, consider the formula. Using Kepler's Harmonic Law $P^2 = \dfrac{a^3}{(m+M)}$ and $G = 4\,\pi^2$, the equation for the unexplained advance becomes

$$\frac{d\omega}{dt} = \frac{24\,\pi^3\,a^2}{c^2(1-e^2)\,P^2} = \frac{6\pi\,G}{c^2} \times \frac{m+M}{a} \times \frac{1}{(1-e^2)}$$

$$= \frac{6\,\pi}{c^2}\,G\,(m+M)\,\frac{1}{2}\left(\frac{1}{a(1-e)} + \frac{1}{a(1+e)}\right)$$

$$= \frac{3\,\pi}{c^2}\left(\frac{1}{M} + \frac{1}{m}\right)\left[\,G\,m\,M\left(\frac{1}{a(1+e)} + \frac{1}{a(1-e)}\right)\right]$$

The quantity within the square brackets is the sum of the gravitational potential energies at perihelion and aphelion. Were it to be increased or decreased by some different form(s) of gravitational energy added to or taken from it, the periastron advance would be correspondingly increased or decreased. What possible sources of gravitational energy should be added or subtracted?

The binary star PSR 1913 +16 gave a dramatic demonstration of how energy may be lost by gravitational radiation (see "Cosmology in Flat Space-Time", p. 418 et seq.) . In this case, the loss of energy reduced the mean distance *a* and the period *P*. Its effect on the periastron advance would also be to decrease it .

The components of this binary system are neutron stars, therefore of very small diameter compared to a normal star. Their contraction from normal size has produced a "spinup" or increase of angular velocity in order to preserve angular momentum. Therefore the rotational kinetic energy of the system has increased substantially, resulting in an increase in the relativistic advance of periastron. These two factors — loss of energy by gravitational radiation and increase of rotational kinetic energy by gravitational contraction — if included in the equation above, would cause the observed anomalies in the periastron advance. There is no way to know their amount without a detailed knowledge of their histories; we must be content simply to understand their source. The predicted rate of advance for Mercury is identical with the observed rate, however, inasmuch as both effects are for all practical purposes zero in this instance. For binary star systems, this need not be so, as the observations show.

The equation also makes clear that it is the variation of Mercury's mass ($\Delta m = \dfrac{\Delta E}{c^2}$) that produces the unaccounted for advance. It is evident that a physical effect (advance of perihelion) should have a physical cause (change of mass of Mercury) , not a mathematical one (curvature of spacetime).

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