Attenuation and the Expansion of the Kinematic Universe

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Consider two "identical clocks", C and $\bar{C}$. According to the Lorentz transformation, if one clock recedes from the other at a constant speed V, the reading t of the stationary clock and the reading $\bar{t}$ of the receding clock will be related by the equation

(1) $$\bar{t} = \frac{t - \frac{Vr}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}}.$$

Now if, as in the expanding universe described by Hubble's Law $r = Vt$, then

(2) $$\bar{t} = \sqrt{1 - V^2/c^2}\,t.$$

This equation is usually interpreted to mean that the receding clock runs more slowly than the stationary one. It would be more accurate to say that the receding clock **appears** to run more slowly than the stationary one. In point of fact, the two identical clocks are running at the same rate, a state which cannot be changed by their relative motion. (Which clock is the one in motion?) To clarify the distinction, take differentials of both sides, Then

(3) $$\Delta\bar{t} = \sqrt{1 - V^2/c^2}\,\Delta t.$$

Since $$\frac{V}{c} = \frac{s^2 - 1}{s^2 + 1},$$

where s is the red shift $s = 1 + z = \frac{\bar{\lambda}}{\lambda}$, this becomes

(4) $$\Delta\bar{t} = \frac{2s}{s^2 + 1}\Delta t = A \times \Delta t.$$

Here A is the **attenuation.**

This term is appropriate since the stream of photons from $\bar{C}$, emitted in 1 second, will at the end of the second be emitted from a greater distance than those photons which were emitted at the beginning of the second. In short, the train of photons arriving at C has been strung out (attenuated) as a result of the recession of $\bar{C}$. The last photons are at too great a distance and at too late a time to be received at C within one second by C's clock. Clearly, when $s = 1$, $A = 1$ and there is no attenuation. But whenever $s > 1$, $A < 1$. To the clock C the clock $\bar{C}$ will appear fainter by the factor A.

Remote rapidly receding objects (e.g., supernovae or galaxies) will appear fainter than otherwise and unless attenuation is allowed for, their luminosity distances will be overestimated. This will falsify the determination of such things as the Hubble constant, hence the rate of expansion in the early universe. The following table shows the effects of attenuation.

s	Attenuation	Mag Corr	Dist Corr	Time t	Time T	Dist d	Dist ρ
1.0	1.000000	0.000000	1.000000	14.00000	14.00000	0.00000	0.00000
1.1	0.995475	-0.004924	0.997735	12.78512	12.66566	1.21488	1.33434
1.2	0.983607	-0.017946	0.991769	11.86111	11.44750	2.13889	2.55250
1.3	0.966543	-0.036947	0.983129	11.14201	10.32690	2.85799	3.67310
1.4	0.945946	-0.060334	0.972598	10.57143	9.289389	3.42857	4.71061
1.5	0.923077	-0.086905	0.960769	10.11111	8.323488	3.88889	5.67651
1.6	0.898876	-0.115750	0.948091	9.73438	7.419949	4.26563	6.58005
1.7	0.874036	-0.146177	0.934899	9.42215	6.571204	4.57785	7.42880
1.8	0.849057	-0.177658	0.921443	9.16049	5.770987	4.83951	8.22901
1.9	0.824295	-0.209793	0.907907	8.93906	5.014046	5.06094	8.98595
2.0	0.800000	-0.242275	0.894427	8.75000	4.295939	5.25000	9.70406
3.0	0.600000	-0.554622	0.774597	7.77778	-1.38057	6.22222	15.38057
4.0	0.470588	-0.818397	0.685994	7.43750	-5.40812	6.56250	19.40812
5.0	0.384615	-1.037433	0.620174	7.28000	-8.53213	6.72000	22.53213
..	..	..	..	..	..	..	..
2000.0	0.001000	-7.500000	0.031623	7.00000	-92.4126	7.00000	106.41263

$A = \dfrac{2s}{s^2 + 1}$, Magnitude correction = $\Delta m = 2.5 \times \log A$,

Distance correction = $\dfrac{d}{d_L} = \sqrt{A}$, kinematic time $t_k = \dfrac{t_0}{2}(1 + \dfrac{1}{s^2})$, dynamical time $T_d = t_0(1 - \ln s)$, kinematic distance $d_k = \dfrac{1}{2} c\, t_{(0)} (1 - \dfrac{1}{s^2})$,

dynamical distance $\rho = c\, t_{(0)} \ln s$

The last line shows the case for the microwave background radiation (MBR). Attenuation diminishes the luminosity of the MBR by 7.5 magnitudes (!), thus identifying the most important component of the Olbers' paradox that the night sky is dark.