

Viva Special Relativity!

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Special Relativity (SR) was put forward in 1905 by Albert Einstein. Parts of the theory had been current for some time, but Einstein drew them all together. The theory dealt with electromagnetism and with the dynamics of inertial systems but seemed powerless to compass the finer points of gravitation. To remedy this deficiency, Einstein put forward in 1915 his General Theory of Relativity (GR). The latter succeeded where the former failed. It has become the unquestioned theory of gravitation whereas the Special Theory is regnant only in electromagnetism.

This dichotomy is somewhat of an embarrassment. Though both gravitational and electrical attractions are described by inverse square laws, electromagnetism includes magnetic interactions of moving charged particles but gravitation appears to have no counterpart. Though electromagnetism implies radiation by accelerated electrical charges, Newtonian gravitation appears not to imply radiation by accelerated masses. Physicists and astronomers have reluctantly accepted this state of affairs; they apply the equations of SR to electrically charged bodies and the equations of GR to uncharged masses. Is this a real and necessary distinction?

A closer inspection of the respective equations shows that their derivations are unlike. Suppose, therefore, that we derive the equations of gravitation in the same way as those of electromagnetism. The results (see “Cosmology in Flat Spacetime”) are now identical in form with those of electromagnetism and do predict correctly gravitational radiation and gravitomagnetism.

So far, so good. Will the new equations also predict gravitational effects such as the advance of the perihelion of Mercury? The answer is “Not yet”. There remains one more necessary modification: inclusion of the special relativistic mass-energy relation, $E = Mc^2$. Specifically, the work done by the sun in bringing Mercury from infinity will increase Mercury’s mass ever so slightly. It will also increase the mass of the sun by the same amount, but only 3.9 millionths of the sun’s mass, a negligible amount. Mercury’s path with a variable mass will not be the same as it would have been with constant mass between perihelion and aphelion. It is this difference which both GR and SR now predict correctly.

Will either GR or SR identify “dark matter”? Despite Herculean efforts, GR has so far been unsuccessful. SR, on the other hand, recognizes both baryonic mass M_b – the sum of the individual masses of all the fundamental particles (protons, neutrons, electrons, etc.) – and M_g , the “gravitational mass”

$$\overline{M_g} = N \times \overline{\mu_j} \left(\overline{\exp N \frac{G}{c^2}} \right) \overline{\exp \frac{\mu_i}{r_{ij}}} = \overline{M_b} \overline{\exp \frac{\mu_i}{r_{ij}}}$$

When N is very large, as it would be for a galaxy or a globular cluster, the averages (indicated by the bars), would have $\frac{1}{2}N(N-1)$ terms, an impossibly large task for available computers. Does this imply that the answer is necessarily unknown? Could these averages be estimated with any confidence?

Perhaps. In a large population, it is not unlikely that one or more members has (have) the value of the average. This is especially true for a large symmetric population such as a globular cluster. For the sake of illustration, consider a system of uniform density. Let all stars have unit mass. Then, along any diameter, every 0.1R

$$\frac{M_g}{M_b} \equiv \exp \left[\frac{1}{20} \left(\frac{1}{2.0} + \frac{1}{1.9} + \frac{1}{1.8} + \dots + \frac{1}{0.3} + \frac{1}{0.2} + \frac{1}{0.1} \right) \right] = \exp 1.7989 = 6.043$$

This crude example demonstrates how the gravitational mass may exceed by a substantial factor the baryonic mass. It is also gratifying that the computed factor is so similar to the observationally determined one. This is not a proof that the preceding argument is correct, but it is a result compatible with it. Viva SR!